Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced Subsidiary

Wednesday 17 June 2009 – Morning

Time: 1 hour 30 minutes

<u>Materials required for examination</u>	<u>Items included with question papers</u>
Mathematical Formulae (Orange)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

M35146A

1. The complex numbers z_1 and z_2 are given by

$$z_1 = 2 - \mathrm{i} \text{ and } z_2 = -8 + 9\mathrm{i}$$

(*a*) Show z_1 and z_2 on a single Argand diagram.

(1)

Find, showing your working,

(*b*) the value of $|z_1|$,

(2)

(*c*) the value of arg z_1, giving your answer in radians to 2 decimal places,

(2)

(*d*) $\dfrac{z_2}{z_1}$ in the form $a + b\mathrm{i}$, where a and b are real.

(3)

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2. (*a*) Using the formulae for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, show that

$$\sum_{r=1}^{n} r(r+1)(r+3) = \frac{1}{12}n(n+1)(n+2)(3n+k),$$

where k is a constant to be found.

(7)

(*b*) Hence evaluate $\sum_{r=21}^{40} r(r+1)(r+3)$.

(2)

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3. $$\mathrm{f}(x) = (x^2 + 4)(x^2 + 8x + 25)$$

(*a*) Find the four roots of $\mathrm{f}(x) = 0$.

(5)

(*b*) Find the sum of these four roots.

(2)

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4. Given that α is the only real root of the equation

$$x^3 - x^2 - 6 = 0,$$

(*a*) show that $2.2 < \alpha < 2.3$

(2)

(*b*) Taking 2.2 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x) = x^3 - x^2 - 6$ to obtain a second approximation to α, giving your answer to 3 decimal places.

(5)

(*c*) Use linear interpolation once on the interval [2.2, 2.3] to find another approximation to α, giving your answer to 3 decimal places.

(3)

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5. $$\mathbf{R} = \begin{pmatrix} a & 2 \\ a & b \end{pmatrix}, \text{ where } a \text{ and } b \text{ are constants and } a > 0.$$

(*a*) Find $\mathbf{R}^2$ in terms of a and b.

(3)

Given that $\mathbf{R}^2$ represents an enlargement with centre (0, 0) and scale factor 15,

(*b*) find the value of a and the value of b.

(5)

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6. The parabola C has equation $y^2 = 16x$.

(*a*) Verify that the point $P(4t^2, 8t)$ is a general point on C.

(1)

(*b*) Write down the coordinates of the focus S of C.

(1)

(*c*) Show that the normal to C at P has equation

$$y + tx = 8t + 4t^3 .$$

(5)

The normal to C at P meets the x-axis at the point N.

(*d*) Find the area of triangle PSN in terms of t, giving your answer in its simplest form.

(4)

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7. $$\mathbf{A} = \begin{pmatrix} a & -2 \\ -1 & 4 \end{pmatrix}, \text{ where } a \text{ is a constant.}$$

(*a*) Find the value of *a* for which the matrix **A** is singular.

(2)

$$\mathbf{B} = \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}$$

(*b*) Find $\mathbf{B}^{-1}$.

(3)

The transformation represented by **B** maps the point *P* onto the point *Q*.

Given that *Q* has coordinates $(k - 6, 3k + 12)$, where *k* is a constant,

(*c*) show that *P* lies on the line with equation $y = x + 3$.

(3)

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8. Prove by induction that, for $n \in \mathbb{Z}^+$,

(*a*) $f(n) = 5^n + 8n + 3$ is divisible by 4,

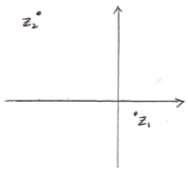
(7)

(*b*) $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^n = \begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix}$.

(7)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)		B1 (1)
(b)	$\lvert z_1\rvert = \sqrt{2^2+(-1)^2} = \sqrt{5}$ (or awrt 2.24)	M1 A1 (2)
(c)	$\alpha = \arctan\left(\frac{1}{2}\right)$ or $\arctan\left(-\frac{1}{2}\right)$	M1
	$\arg z_1 = -0.46$ or 5.82 (awrt)	A1 (2)
(d)	$\frac{-8+9\mathrm{i}}{2-\mathrm{i}} \times \frac{2+\mathrm{i}}{2+\mathrm{i}}$	M1
	$= \frac{-16-8\mathrm{i}+18\mathrm{i}-9}{5} = -5+2\mathrm{i}$ i.e. a = -5 and b =2 or $-\frac{2}{5}a$	A1 A1ft (3)
		(8 marks)
2. (a)	$r(r+1)(r+3) = r^3+4r^2+3r$, so **use** $\sum r^3 + 4\sum r^2 + 3\sum r$	M1
	$= \frac{1}{4}n^2(n+1)^2 + 4\left(\frac{1}{6}n(n+1)(2n+1)\right) + 3\left(\frac{1}{2}n(n+1)\right)$	A1 A1
	$= \frac{1}{12}n(n+1)\{3n(n+1)+8(2n+1)+18\}$ or $= \frac{1}{12}n\{3n^3+22n^2+45n+26\}$	
	or = $= \frac{1}{12}(n+1)\{3n^3+19n^2+26n\}$	M1 A1
	$= \frac{1}{12}n(n+1)\{3n^2+19n+26\} = \frac{1}{12}n(n+1)(n+2)(3n+13)$ (k = 13)	M1 A1cao (7)
(b)	$\sum_{21}^{40} = \sum_{1}^{40} - \sum_{1}^{20}$	M1
	$= \frac{1}{12}(40\times41\times42\times133) - \frac{1}{12}(20\times21\times22\times73) = 763420 - 56210 = 707210$	A1 cao (2)
		(9 marks)

Question Number	Scheme	Marks
3. (a)	$x^2 + 4 = 0 \quad \Rightarrow \quad x = k\text{i}, \quad x = \pm 2\text{i}$	M1, A1
	Solving 3-term quadratic	M1
	$x = \dfrac{-8 \pm \sqrt{64-100}}{2} \quad = -4+3\text{i}$ and $-4-3\text{i}$	A1 A1ft (5)
(b)	$2\text{i} + (-2\text{i}) + (-4+3\text{i}) + (-4-3\text{i}) = -8$	M1 A1cso (2)
		(7 marks)
4. (a)	$\text{f}(2.2) = 2.2^3 - 2.2^2 - 6 \quad (= -0.192)$ $\text{f}(2.3) = 2.3^3 - 2.3^2 - 6 \quad (= 0.877)$	M1
	Change of sign $\Rightarrow$ Root need numerical values correct (to 1 s.f.).	A1 (2)
(b)	$\text{f}'(x) = 3x^2 - 2x$	B1
	$\text{f}'(2.2) = 10.12$	B1
	$x_1 = x_0 - \dfrac{\text{f}(x_0)}{\text{f}'(x_0)} = 2.2 - \dfrac{-0.192}{10.12}$	M1 A1ft
	$= 2.219$	A1cao (5)
(c)	$\dfrac{\alpha - 2.2}{\pm '0.192'} = \dfrac{2.3-\alpha}{\pm '0.877'}$ (or equivalent such as $\dfrac{k}{\pm '0.192'} = \dfrac{0.1-k}{\pm '0.877'}$.)	M1
	$\alpha(0.877+0.192) = 2.3 \times 0.192 + 2.2 \times 0.877$ or $k(0.877+0.192) = 0.1 \times 0.192$, where $\alpha = 2.2 + k$	A1
	so $\alpha \approx 2.218$ (2.21796…) (Allow awrt)	A1 (3)
		(10 marks)
5. (a)	$\mathbf{R}^2 = \begin{pmatrix} a^2+2a & 2a+2b \\ a^2+ab & 2a+b^2 \end{pmatrix}$	M1 A1 A1 (3)
(b)	Puts their a^2+2a = 15 or their $2a+b^2$ = 15 or their $(a^2+2a)(2a+b^2)-(a^2+ab)(2a+2b)$ = 225 (or to 15) ,	M1,
	Puts their $a^2+ab=0$ or their $2a$ + $2b$ = 0	M1
	Solve to find either a or b	M1
	$a=3, \quad b=-3$	A1, A1 (5)
		(8 marks)

Question Number	Scheme	Marks
6. (a)	$y^2 = (8t)^2 = 64t^2$ and $16x = 16\times 4t^2 = 64t^2$ Or identifies that a = 4 and uses general coordinates $(at^2, 2at)$	B1 (1)
(b)	(4, 0)	B1 (1)
(c)	$y = 4x^{\frac{1}{2}}$ $\frac{dy}{dx} = 2x^{-\frac{1}{2}}$	B1
	Replaces x by $4t^2$ to give **gradient** $[\,2(4t^2)^{-\frac{1}{2}} = \frac{2}{2t} = \frac{1}{t}\,]$	M1,
	Uses Gradient of normal is $-\frac{1}{\text{gradient of curve}}$ $[-t]$	M1
	$y - 8t = -t(x - 4t^2)$ $\Rightarrow$ $y + tx = 8t + 4t^3$ (*)	M1 A1cso (5)
(d)	At N, y = 0, so $x = 8 + 4t^2$ or $\frac{8t+4t^3}{t}$	B1
	Base $SN = (8+4t^2) - 4$ $(= 4 + 4t^2)$	B1ft
	Area of Δ PSN = $\frac{1}{2}(4+4t^2)(8t) = 16t(1+t^2)$ or $16t + 16t^3$ for $t > 0$	M1 A1 (4)
		(11 marks)
7. (a)	Use $4a - (-2\times -1) = 0$ $\Rightarrow$ $a, = \frac{1}{2}$	M1, A1 (2)
(b)	Determinant: $(3\times 4) - (-2\times -1) = 10$ (Δ)	M1
	$\mathbf{B}^{-1} = \frac{1}{10}\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$	M1 A1cso (3)
(c)	$\frac{1}{10}\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} k-6 \\ 3k+12 \end{pmatrix}, \quad = \frac{1}{10}\begin{pmatrix} 4(k-6)+2(3k+12) \\ (k-6)+3(3k+12) \end{pmatrix}$	M1, A1ft
	$\begin{pmatrix} k \\ k+3 \end{pmatrix}$ Lies on $y = x + 3$	A1 (3)
		(8 marks)

Question Number	Scheme	Marks
8. (a)	$f(1) = 5+8+3=16$, (which is divisible by 4). ($\therefore$ True for n = 1).	B1
	Using the formula to write down f(k + 1), $f(k+1) = 5^{k+1} + 8(k+1) + 3$	M1 A1
	$f(k+1) - f(k) = 5^{k+1} + 8(k+1) + 3 - 5^k - 8k - 3$	M1
	$= 5(5^k) + 8k + 8 + 3 - 5^k - 8k - 3 = 4(5^k) + 8$	A1
	$f(k+1) = 4(5^k + 2) + f(k)$, **which is divisible by** 4	A1ft
	$\therefore$ True for n = k + 1 **if** true for n = k. True for n = 1, $\therefore$ true for all n.	A1cso (7)
(b)	For n = 1, $\begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^1$ **(**$\therefore$ True for n = 1.)	B1
	$\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$	M1 A1 A1
	$= \begin{pmatrix} 2(k+1)+1 & -2(k+1) \\ 2(k+1) & 1-2(k+1) \end{pmatrix}$	M1 A1
	$\therefore$ True for n = k + 1 **if** true for n = k. **True for *n*** = 1, $\therefore$ **true for all *n***	A1 cso (7)
		(14 marks)